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Approximate Determination of the Shape of Solidified Materials with Either Shrinkage or Expansion

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(Received January 18, 1995)

The differences in densities between liquid and solid phases results in a concave surface during solidification. The final shape after complete solidification is obtained by considering the conservation of the nass without any reference to the detailed transport processes. An easy and quick procedure is described to predict the final shape of the solid phase for planar and cylindrical configurations. The results are validated by comparison with an existing data.

Key Words: Density Difference, Solidification, Shrinkage, Expansion, Phase Change

Nomenclature

A = area
ε = parameter, $\rho_S/\rho_L - 1$
ρ = density

Subscripts

L = liquid phase
S = solid phase

1. Introduction

There are vast number of applications involving the solid/liquid phase transition and considerable amount of efforts have been devoted to investigating the phase-change phenomenon in almost all its aspects: microsegregation (Battle and Pehlke, 1990), void formation (Sulfredge, Chew and Tagari, 1992) and influence of natural convection (Viskanta, 1983), to name but a few. In particular, the density difference between phases has increasingly attracted experimental (Sparrow and Broadbent, 1982; Ho and Viskanta, 1983; Ho and Viskanta, 1984) and analytical (Shamsundar and Sparrow, 1976; Kim, Ro, Lee and kim, 1993) research attention. Whereas these studies primarily concerned with the *transient* and *local* Characteristics of the heat transfer, this note aims at presenting a rather different view of the phasechange process by focusing only on the final shape after complete solidification with density difference.

The difference in densities is of importance especially in static casting processes for an overall design of the size and shape of the mold (Flemings, 1974), because phase-change materials with a few exceptions normally shrink during solidification and hence the detachment of the liquid from the mold wall results in a concave shrinkage surface due to gravity. The focus here is on the description of an easy and quick procedure to predict the final shape of the solid-phase when the solidification is completely achieved. The strategy to fulfil this objective is guided by the conservation of the total mass without any reference to the detailed transport processes. The validity of the present approach is then assessed by comparing with an existing work with density difference.

2. Simplified Analysis

Figure 1 illustrates an idealized situation at an intermediate stage of the solidification process with volume contraction in a two-dimensional rectangular enclosure subject to cooling from the vertical sides and insulation otherwise. The exact elapsed time to arrive at that configuration is out

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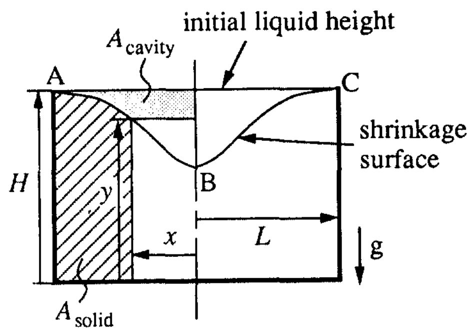

Fig. 1 Schematic representation of the shrinkage surface due to volume contraction in a two-dimensional rectangular enclosure.

of the scope in this note. Instead, the primary concern is on the determination of the shape of the shrinkage surface. To facilitate the analysis, simplifications are introduced such as no encaptured void inside both phases, the perfect flatness of the liquid free surface and the movement of the interface parallel to the wall, as shown in the figure.

At any point on the curve ABC that designates the top shrinkage surface of the completely solidified material, the conservation of the total mass is simply expressed as

$$(\rho_S - \rho_L) A_{solid} = \rho_L A_{cavity} \tag{1}$$

where A_{solid} and A_{cavity} are evaluated from the following:

$$A_{solid} = \int_x^L y\,dx,\ A_{cavity} = \int_y^H x\,dy. \tag{2}$$

Any different mode of the interface movement can be readily accommodated by modifying Eq. (2) appropriately. For more tractable solution, an alternative expression of Eq. (1) is preferred here in the following differential form:

$$(\rho_S - \rho_L) y\,dx = \rho_L x\,dy. \tag{3}$$

By introducing a density ratio parameter, $\varepsilon = \rho_S/\rho_L - 1$, and by integrating Eq. (3) with the boundary condition, $y = H$ at $x = L$, we obtain

$$\frac{y}{H} = \left(\frac{x}{L}\right)^{\varepsilon},\ 0 < x \leq L \tag{4}$$

Figure 2 displays a few curves obtained from Eq. (4) by varying ε. Table 1 displays values of ε for sevaral materials. The case of $\varepsilon = 0$ refers to that of no density difference between phases and hence

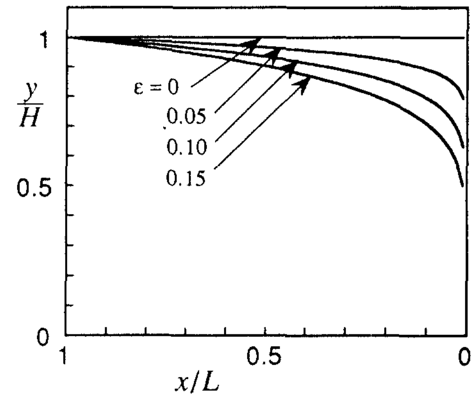

Fig. 2 Normalized shapes of the shrinkage surface versus the parameter ε for the case corresponding to Fig. 1.

Table 1 Values of ε for Several materials

material	values of ε
Aluminum	0.0707
Copper	0.0515
Magnesium	0.0438
Iron	0.0417
N-Octadecane	0.0496
Water	−0.0829

* (Flemings, 1974)

corresponds to a horizontal curve in the figure. Equation (4) implies that, when the shrinkage surfaces obtained experimentally by varying heights and base-lengths are normalized, they would collapse onto a curve with slight deviations. (The expected small deviation would depend on the initial superheat, the aspect ratio, the rate of freezing, etc.) For paraffins being widely used in experiments, its value of ε is about ~0.1 indicating that its final solidified shape will show a high concavity. In such cases, the present analysis can be utilized to foretell the solidification behavior as a first approximation.

Next, consider the inward solidification process in a cylindrical enclosure cooled from the side and insulated in the top and bottom surfaces. Referring to Fig. 3 and assuming the interface to move parallel to the vertical as before, the constraint of the total mass conservation reads

$$\varepsilon \cdot 2\pi r z\,dr = \pi r^2 dz \tag{5}$$

which can be immediately integrated with the proper boundary condition (i.e. $z=H$ at $r=R$) to yield

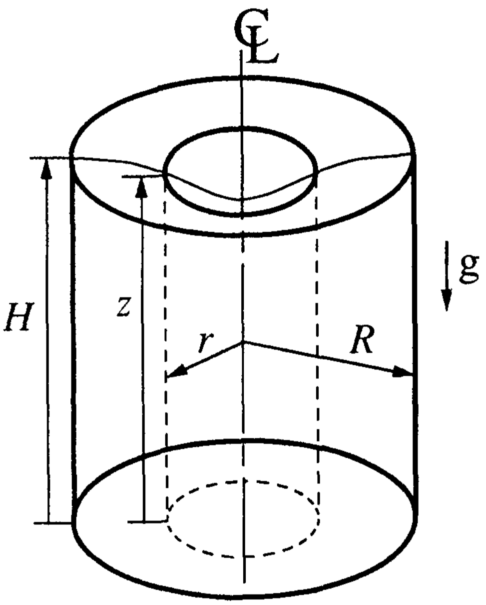

Fig. 3 Schematic representation of the shrinkage surface due to volume contraction in a cylindrical enclosure.

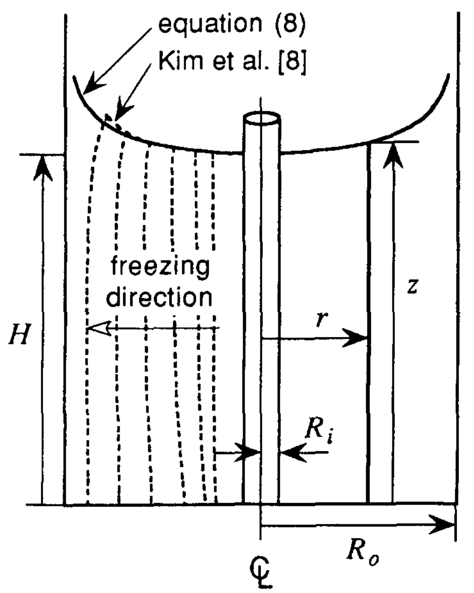

Fig. 4 Schematic diagram for the outward freezing of water filled between vertical concentric tubes (right half) and the normalized shape of the ice top surface (left half).

$$\frac{z}{H}=\left(\frac{r}{R}\right)^{2\varepsilon} \tag{6}$$

which is very similar to Eq. (4) For further illustration of the present approach, the outward freezing of water filled between vertical concentric tubes (Kim, Ro, Lee and Kim, 1993) is schematically shown in the right half of Fig. 4. Since the ice is at a density lower than the water, the complete solidification creates the upward bulged surface of the ice, as shown in Fig. 4. By assuming no overflow of the water and the movement of the interface parallel to the vertical, the total mass conservation can be written as

$$-\varepsilon \cdot 2\pi rz\mathrm{d}r=\pi(R_0^2-r^2)\mathrm{d}z \tag{7}$$

which can be integrated with the boundary condition, $z=H$ at $r=R_i$, to yield

$$\frac{z}{H}=\left(\frac{R_0^2-r^2}{R_0^2-R_i^2}\right)^{2\varepsilon} \tag{8}$$

Using $\varepsilon=-0.0829$ for the ice-water system, the above equation is plotted in the left half of Fig. 4 where the numerical results (see Fig.7(a) in ref. (Kim, Ro, Lee and Kim, 1993) are also shown for comparison. The vertical dotted lines represent the loci of the interface at several times that were obtained from the solution of both the conduction in the ice and the natural convection in the water. Although the present analysis considers only the mass conservation, good agreement with rigorous analysis is evident in the figure. However, since the presumed mode of the interface movement is not realistic near $r=R_0$, notable disagreement can be expected there. Nevertheless, fairly good agreement over the range considered seems to be encouraging to remark the utility of the present approach.

3. Concluding Remarks

The final shape after complete solidification was predicted by considering only the conservation of the total mass. Analysis was performed for the planar and cylindrical configurations and validated by comparison with an existing study with density difference. When the mode of the interface movement is properly taken into consideration, the approach is capable of accommodating three-dimensional or other complex geometries as well. The present approach would be useful in cases the final shape of the solidified material is by itself important and a rigorous analysis appears extraordinarily complicated.

Acknowledgement

The work was supported by Turbo and Power Machinery Research Center.

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